Celebrating Sixty
30 Problems and 30 Solutions

1. a. How many seconds in a minute?
   b. How many minutes in an hour?
   c. How many hours in a day?.....OK, in two and a half days?
2. If today is Friday, what day will it be 60 days from now?
3. What is the sum of the first 60 counting numbers $1 + 2 + 3 + \cdots + 60$?
4. a. List all the factors of 60.
   b. What is the sum of all these factors?
   c. What is the prime factorization of 60?
5. The first Western Mathematics Teachers Conference occurred in 1951.
   a. What are the common factors of 1951 and 2011?
   b. Insert some of the basic operations $\{+, -, \times, \div\}$ between some of the digits of 1951 to make the following a true statement
   $$1951 = 60.$$
6. What is the units digit of 2^{60}?
7. How many different "words" can be created using all the letters of SIXTY?
8. How many different "words" can be created using all the letters of ISATS?
9. Sarah has some marbles. If she divides them into 2 equal groups, there is exactly one left over. The same is true when she divides them into $3, 4, 5$, or 6 equal groups. What is the least number of marbles Sarah could have?
10. What is the degree measure of each interior angle of a regular polygon with 3 sides?
11. What is the degree measure of each interior angle of a regular polygon with 60 sides?
12. A rectangular garden has an area of 60 square feet. If its dimensions are integer values, what possible perimeters could this garden have?
13. A farmer has 60 feet of fence with which to build a rectangular pen. What would be the maximum area possible for such a pen?
14. The consecutive integers $1, 2$, and 3 are factors of 60 and have the property that their sum $(1 + 2 + 3)$ is equal to their product $1 \cdot 2 \cdot 3$. Determine three other consecutive integers that have this same property.
15. Determine $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$.

16. a. Snooki, Izzy, Xavier, Ted, and Yani stand in a circle in the order they are listed. Snooki begins counting by saying 1, Izzy follows with 2 and so forth. When a person says an *odd* number, they are eliminated (so, yes, Snooki is eliminated right away) and the count continues with the next person. When these five people play, who will be the last person standing?

    b. If this game were played with 60 people, starting with player 1, 2, etc., which player would be the last one standing?

    c. Can you describe a general way to determine the last person standing when any number N people play the game?

17. a. Suppose letters in the alphabet were assigned numbers such that $A = 1$, $B = 2$, $C = 3$, and so forth. What would be the sum of the numbers represented by SIXTY?

    b. Using this system, determine a word that has a sum value of 60.

18. Find the last two digits of the number $S = 1! + 2! + 3! + \cdots + 60!$.

19. The last n digits of the number 60^{60} are all zeroes. Find n.

20. Find the smallest positive integer that has exactly 60 divisors.

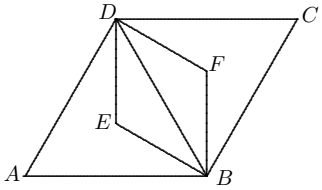
21. If the area of an equilateral triangle is 60 square inches, what is its perimeter?

22. If the perimeter of an equilateral triangle is 60 inches, what is its area?

23. Find the smallest positive integer larger than 2 that leaves a remainder of 2 when divided by each of the numbers 3, 4, 5 and 6.

24. How many 6 by 6 squares can be formed on a 60 by 60 grid?

25. Rhombus $ABCD$ is similar to rhombus $BFDE$. The area of rhombus $ABCD$ is 60, and $\angle BAD = 60^\circ$. What is the area of rhombus $BFDE$? (AMC 2006)



26. For each positive integer n, the mean of the first n terms of a sequence is equal to n. What is the 60th term of the sequence? (AMC 2008)

27. Let A, B and C be three distinct points on the graph of $y = x^2$ such that line AB is parallel to the x-axis and $\triangle ABC$ is a right triangle with area 60. What is the sum of the digits of the y-coordinate of C? (AMC 2008)

28. Suppose that a, b, and c are positive real numbers such that $a^{\log_5 2} = 125$, $b^{\log_2 7} = 4$, and $c^{\log_7 9} = \sqrt{7}$. Find $a^{(\log_5 2)^2} + b^{(log_2 7)^2} + c^{(\log_7 9)^2}$. (AIME 2009)

29. Use the fact that there are 60 minutes in an hour to find the angle between the hour and minute hands on a clock at

Easy:	a) 3:00	b) 1:00
Medium:	c) 8:30	d) 1:20
Harder:	e) 2:18	f) 5:24

30. If 60 cats can catch 60 mice in 60 minutes, how many cats are needed to catch 100 mice in 100 minutes?